The Method of Arbitrary Functions

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1. Introduction

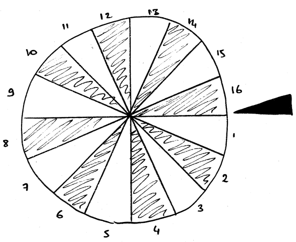
There has been widespread exitement in the literature about the capacity of the *method of arbitrary functions* to reconcile non-trivial objective probabilities and deterministic mechanics.

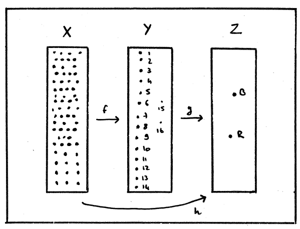
> **Butterfield (2011)** [the method is] very suggestive for the philosophy of probability. [It] hints that even with an underlying determinism, [one] can define non-trivial probabilities that are "objectively correct". (p. 1084)
>
> **Myrvold (2012)** In situations like [those considered by the method of arbitrary functions], we are justified, I think, in calling this common probability the *chance*. (p. 80)
>
> **Suarez (forth.)** [the method shows that] objective chance is a dynamical epi-phenomenon of complexity—quite independently of whether the underlying dynamics is deterministic or not.
>
> **Strevens (2011)** [the method] has the potential to account for all the deterministic physical probability to be found in our world. (p. 364)
>
> **von Plato (1983).** It has been used to justify a unique and objectively interpreted concept of probabiltity in games of chance of a mechanical type, and in the theory of dynamical systems of mathematical physics. (p. 37)

Our plan for the talk: introduce the method (§2), discuss what sort of philosophical significance it could have (§3), and look at the two kinds in turn (§4–5).

2. The Method of Arbitrary Functions

General idea. Suppose that you are faced with a wheel, painted with alternating black and red wedges of equal size, equipped with a stationary pointer. The wheel may be spun, and allowed to come at a stop with the pointer indicating either red or black—call this a *trial.* Suppose further that the mechanics of the wheel are deterministic, and that the outcome of a given trial is fixed by a single parameter, the initial speed of the wheel. The method of arbitrary functions supposedly enables us to establish that the probability of the wheel coming to a stop on a red wedge is equal to the probability of the wheel coming to a stop on a black wheel; where the probabilities are interpreted objectively.





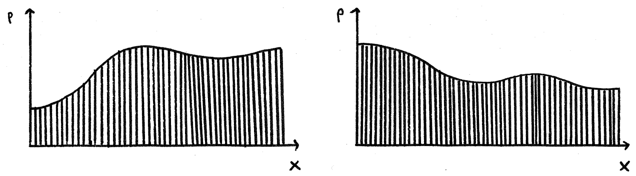
Probability. We define a probability space $\langle X, \mathcal{A}, p_i \rangle$ such that f and h are random variables. Now, if p_i is a uniform distribution over Y, then it's a uniform distribution over Z. Two possible arguments:

> **Direct argument.** Three problems with a principle of indifference: Bertrand's paradox, severe uncertainty, dependence on interpretation.
>
> **Indirect argument.** We have no reason to expect p_i is uniform over X (severe uncertainty + interpretation), or that f maps the elements of X to those of Y in equal proportions (severe uncertainty).

Method of Arbitrary Functions. Take any p_i, if the two following plausible assumptions are satisfied, then p_i is roughly uniform over Z: $p_i(R) \approx p_i(B)$.

(i) There are many more possible initial velocities than there are colours ($|X| >> |Z|$).

(ii) The probability function p_i is relatively "smooth" in the sense that two very similar initial impulses get roughly the same probability.



3. Philosophical Significance

The philosophical question. The method of arbitrary functions tells us that there is a set $\{p_i\}$ of probability functions over $\langle X, \mathcal{A}\rangle$, such that $p_i(\{B\}) \approx p_i(\{R\})$ for all i. So, we can define a new probability function, P, which is defined on a subset of $\mathcal{A}$, call it $\mathcal{A}_Z = \{X, \emptyset, X_B, X_R\}$ where $X_B = \{x \in X : h(x) = B\}$ and $X_R = \{x \in X : h(x) = R\}$; and $P(X_B) = P(X_R) = 1/2$. So, we now have a new probability triple: $\langle X, \mathcal{A}_Z, P\rangle$. How should we interpret this new triple?

Two options. $P = h^{-1} \circ p_i$. Remember that h maps every initial speed to the colour of the outcome of the trial. In those cases that we are considering, where $|X| >> |Z|$, we presumably do not know the specific shape of h. However, it is, by stipluation, objective. The colour of the wedge that will obtain if I spin the wheel at a certain speed is completely agent-independent. This means that there are two cases of interest, for which we ask: is P objective? And, does the set up correctly model games of chance (and statistical mechanical phenomena)?

1. h and p_i are both objective. (§4)
2. h is objective and p_i is subjective. (§5)

4. Objective Probability Input

Comment on objective interpretations. Chance, propensity, and best systems won't do the trick. We should follow Strevens (2011) in using a *frequency* interpretation for p_i. Now, we have the choice between finite and infinite frequentism. I leave the latter to the side because of interpretational issues. In any case I think that some of the problems I raise for finite frequentism plague the infinite case too.

Objections.

(i) The constraints on the set up cease to be trivial: the method will only work in case a wheel is *actually* spun a large amount of times, with a wide and smooth variation over initial conditions.

(ii) The method tells us that, if the constraints are respected, the function P is uniform. So, given an epistemic closure principle, if I know that the constraints are respected, I know that P is uniform. But the antecedent doesn't hold in general, so the method cannot fulfill its putative philosophical purpose.

5. Subjective Probability Input

Subjective interpretations. Myrvold (2012) distinguishes between radical subjectivists, objective Bayesians, and tempered personalists. Let's for now go with him and be tempered personalists.

P **is subjective.** If p_i is subjective, then the constrainsts are plausible. But P in that case is resolutely mind-dependent—it is not objective. That doesn't mean that it's not robust or grounded in facts (Norton and Schaffer). This means that it's practically useful: it tells us for which experiments we can be relaxed about initial conditions.

Philosophically uninteresting. There is no distinction between the method of arbitrary functions and the checkerboard case. A ball is placed on a checkerboard. You can vaguely distinguish which bit of the checkerboard the ball is on, so your credence function over possible initial positions favours and disfavours large areas. Then, the deterministic dynamics of the game are such that, if the ball was on a black square, it is put in a black bowl, and if it was in a white square, it is put in a white bowl. In that case, whatever your credence function, so long as it discriminates equally black and white initial squares, will yield equal values for the ball ending up in the black or the white ball. This is almost trivial!

Circularity. I want to bring a charge of circularity against all of these (modest) conclusions. Myrvold assumes to be tempered personalists. The reason that he provides for being a tempered personalist is that it would be crazy to assign distinct credences to very similar very small-scale initial conditions. But he also uses the assumption of tempered personalism to argue for the plausibility of the constraint. So the question of whether the modest conclusion holds is essentially equivalent to the problem of the priors.

References

Butterfield, J. (2011). Less is Different: Emergence and Reduction Reconciled. *Foundations of Physics 41*(6), 1065–1135.

Myrvold, W. C. (2012). Deterministic Laws and Epistemic Chances. In Y. Ben-Menahem and M. Hemmo (Eds.), *Probability in Physics*, pp. 73–85. Springer.

Schaffer, J. (2007). Deterministic Chance? *British Journal for the Philosophy of Science 58*(2), 113–140.

Strevens, M. (2011). Probability Out Of Determinism. In C. Beisbart and S. Hartmann (Eds.), *Probabilities in Physics*, pp. 339–364. Oxford University Press.

von Plato, J. (1983). The Method of Arbitrary Functions. *British Journal for the Philosophy of Science 34*(1), 37–47.